17 - Apr - 2018 Complexity of an Algorithm.

$O(n^2 + 3n + 1) = n^2$. what does it mean?

$\exists$ Fixed n_0 and fixed ~~a~~ constant c ($\in N$) s.t

$$|n^2 + 3n + 1| \leq cn^2, \quad \forall n \geq n_0 \quad n \in N^*$$

Eg: $O(\sqrt{n} + n) = n$.

$\exists\ n_0$ and c s.t $\quad |\sqrt{n} + n| \leq cn \quad \forall n \geq n_0$

Definition: Polynomials, $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

where $a_0, a_1, \dots a_n$ are $\mathbb{R}$

and all $n \in N$.

Eg: $2x^3 + 7x + 2$ is a polynomial of degree 3.

$\frac{1}{2x^3 + 2x}$ is not a polynomial.

Fact: $O(\text{polynomial}) = n^{\text{highest } \cancel{\text{deg}} \text{ exponent}}$

$= n^{\text{degree}}$

Eg. $O(2x^3 + 10x^5 - 7x + 10) = \cancel{x}^5$

Definitions: Lets call $2x^{3/2} + x^{1/2} + x + 2$ mini-polynomials

i.e looks like polynomial but all exponents are positive rational numbers.

Fact: $O(\text{mini polynomial}) = n^{\text{highest exponent}}$

$= n^{\text{degree}}$

Eg: $O(2n^{3/2} + n^{1/2} + n + 2) = n^{3/2}$

19-Apr-2018: Observe: eg: $n^2 + 3n + 7 \leq n^2 + 3n^2 + 7n^2$

$\leq 11n^2 \quad \forall n \in N$.

Facts: $O(f_1 + f_2) = \max\{O(f_1), O(f_2)\}$

$O(f_1 \cdot f_2) = O(f_1) \cdot O(f_2)$.

$O\left(\frac{f_1}{f_2}\right) = \frac{O(f_1)}{O(f_2)}$

mini-function = $\frac{\text{mini polynomial}}{\text{mini polynomial}}$

rational function = $\frac{\text{polynomial}}{\text{polynomial}}$

Hence $O(\text{mini-function}) = \frac{O(\text{mini polynomial})}{O(\text{mini polynomial})}$

and $O(\text{rational}) = \frac{O(\text{polynomial})}{O(\text{polynomial})}$.

eg: $O\left(\frac{\sqrt[4]{n^3} + \sqrt{n} + 3n}{n^{1/3} + n^{1/5} - 2}\right) = \frac{O(\sqrt[4]{n^3} + \sqrt{n} + 3n)}{O(n^{1/3} + n^{1/5} - 2)} = \frac{n}{n^{1/3}} = n^{2/3}$

Example: For $i=2$ to $(3n+1)$
$x = a*b + 1$
For $k=1$ to i
$y = x \div 3 + b^2 - 1$
next k
next i.

i) Find the exact number of computation that is executed by the code?
ii) Find complexity of code.

Outer loop will be executed (terminal − initial + 1) times
i.e $(3n+1) - 2 + 1$
$= 3n$ times

let L be number of times outer loop executed.
$L: 1, 2, \cdots, 3n$ times

when $L=1$, $i=2$
no. of operations in outer loop = 2.
no. of iterations in inner loop * no. of op. in inner loop = $2 * 4$.

$\therefore$ total no. of op. at $L=1$; $\Rightarrow 2 + 2(4) = 10$

total no. of op. at $L=2 \Rightarrow 2 + 3(4) = 14$

$\vdots$

total no. op. at $L = 3n \Rightarrow 2 + (3n+1)(4)$
$= 2 + 12n + 4$
$= 12n + 6$.

$\therefore$ total no. of op. executed $= \sum_{L=1}^{3n}$ total no. op at L

$$= \frac{(12n+6+10)\,3n}{2}$$

$$= \frac{36n^2 + 48n}{2} = 18n^2 + 24n.$$

ii) Complexity = n^2

$L: 1$ $3n$

outer loop = 2 op

$2 + (2)(4)$ $2 + (3n+1)(4)$

$$\Sigma = \left(\frac{10 + 2 + 12n + 4}{2}\right) 3n$$

$$= (8 + 6n)\,3n$$

$$= 18n^2 + 24n.$$

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Exam II: MTH 213, Spring 2018

$\frac{52}{52}$ Excellent

Ayman Badawi

(Bonus), (1 point). This is math 213 and my instructor name is Ayman Badawi. His office hours are on 4h oh. from to . We meet every, Sunday, Tuesday, and Thursday in Nab at am.

QUESTION 1. Imagine: then

(i) $O(\frac{x^{0.7}+3x^{0.2}-5}{x+7})$ equals to $= \frac{O(x^{0.7}+3x^{0.2}-5)}{O(x+7)} = \frac{x^{0.7}}{x} = x^{-0.3}$

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(ii) $O(\sqrt[3]{x}+x)(x^2-x^{7/2}-2)$ equals to $= O(x^{1/3}+x) \cdot O(x^2 - x^{3.5} - 2)$

$= x \cdot x^{3.5} = x^{4.5}$

QUESTION 2. Imagine: The following Algorithm segment. Find the exact number of additions, multiplications, and subtractions that will be performed when the algorithm is executed. Then find the complexity of the Algorithm segment.

$m = 7; s = 9$

For $k := 3$ *to* (n^2+2)

$L = k * m + 2 * s - 6$ 4 op.

For $i := 1$ *to* $(k+1)$

$s = s + m^3 + i - k^2$ 6 op.

$s + m*m*m + i - k*k$

next i

next k

No. of outer loop iterations #

$L = n^2+2-3+1 = n^2$.

at $L = 1$. ($k = 3$)

outer loop op: 4

inner loop op $= (6)(k+1-1+1)$

$= (6)(4) = 24$.

total op $= 4 + 24 = 28$.

at $L = n^2$ ($k = n^2+2$)

outer loop op $= 4$

inner loop op $= (6)(k+1)$

$= (6)(n^2+2+1)$

$= (6)(n^2+3)$

$= 6n^2+18$.

Total op: $4 + 6n^2 + 18$

$= 6n^2 + 22$.

$\therefore$ Sum $= \frac{(28 + 6n^2 + 22)n^2}{2}$

$= \frac{(6n^2+50)n^2}{2}$

$= 3n^4 + 25n^2$.

Complexity:

$O(3n^4 + 25n^2) = n^4$.

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